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Analysis of moisture diffusivity of larch timber during convective drying condition by using Crank's method and Dincer's method

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Abstract: Two analytical procedures (Crank's method and Dincer's method) for porous solid materials were reevaluated and used to determine moisture diffusion coefficients and moisture transfer coefficients for larch lumber subjected to drying. A diffusion-like equation was used to describe drying process data. The lumber was idealized in the modeling as infinite plates. The moisture transport process inside the board was assumed to be one-dimensional. The macroscopic drying kinetics curves of larch timber at particular conditions were determined experimentally. Based on these data, calculation for both the moisture diffusion coefficients and moisture transfer coefficients by the Dincer's analytical procedure were made. The dynamic moisture diffusion coefficients by the traditional Crank's method were calculated. In general, diffusion coefficients calculated by the Dincer's method were all higher than those by Crank's method. These results could be due to the differences between two analytical methods and also different characteristics between solid moisture diffusion process and heat transfer process. Therefore the analysis and solution procedures of moisture diffusion differential equations need to be adapted in the future. With drying temperature's increasing moisture diffusion coefficient (D) and moisture transfer coefficient (k) increases accordingly. Also the relationships between diffusion coefficients and temperature as well as material moisture contents were analyzed by using *Arrhenius* equation and bound water transport theory.

Keywords: Larch timber; Wood dry; Moisture diffusion coefficient; Moisture transfer coefficient; Mathematical model

Introduction

Kiln drying is a traditional operation method which has wide-range application in China wood industry; however, it is still difficult to analyze this industrial process in a satisfactory way. Wood drying is a process of simultaneous heat and mass transfer, and the mechanism of moisture transfer in wood is very complex due to material biological structure and variable drying medium parameters (temperature, humidity, velocity, etc). Many mathematical models can be used to simulate this drying process. The diffusion coefficient method provides a generalized way that works well for boards with moisture content below the fiber saturation point (FSP), (Simpson 1993; Miao *et al*. 2002; Cai 2005). With diffusion approach, the moisture content gradients during drying process as well as the influences of process parameters and material parameters on drying curve can be estimated, so that the goal of optimizing technical process and saving energy can be achieved to some extent.

The main objectives of this research are: (1) to measure drying kinetics curves of larch lumber(*Larix gmelinii*) under particular experiment conditions; (2) to determine diffusion coefficients and transfer coefficients with Dincer's diffusion analysis; (3) to determine diffusion coefficients as a function of moisture content by using Crank's approach. Further objectives are to compare two kinds of information, illustrate the relationship between model predictions and variations in drying conditions and material characteristics.

Experimental material and procedure

Green larch lumbers were first put in a conditioning chamber with the temperature of (40±2) ºC and a relative humidity of (96±2)%. After the conditioning parameters reached equilibrium, wood moisture contents in thickness direction of lumbers were kept below wood FSP. In other experiment chambers, the temperature was kept at 50, 60, 70, 85ºC, separately, with relative humidity of (50±2)%, and air velocity was 0.5 m/s. Larch lumber was flat-sawn, free of visual defects, four to eight specimens were chosen for each test with specimens dimensions in 200 mm (width) ×150 mm (length) ×20 mm (thickness). Before experiments all specimens' four edges were coated with two layers of epoxy to restrict moisture movement from four edge position, so that all diffusion activities were in wood transverse-radial direction.

Moisture diffusion test was conducted in experimental conditioning chambers, with control sensitivity temperature at ±1ºC, and ±2% of relative humidity. A digital balance with sensitivity of 0.001 g and a weighing sensor with sensitivity of 0.1 g were used for the test. During experiments air temperature, specimen's temperature, wet-bulb temperature, weight of all specimens could be recorded by a computer. After the specimens' moisture distribution came to equilibrium with that of drying media conditions, the specimens were dried at (103±2) ºC to constant weight, so that the oven-dry weight could be obtained. Before the analysis the following assumptions should be made:

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1. The diffusion coefficient is constant in particular moisture content intervals, and the diffusion processes are one dimensional, mainly in wood radial direction.
2. The initial values of temperature and moisture content are uniform within the specimens.
3. The temperature and relative humidity outside surface of the specimens comes to equilibrium with that of the surrounding immediately.
4. The transport of heat, moisture, or gas proceeds in a manner which is symmetrical in space through out the bodies.

Diffusion model solution procedure of two analysis methods

Dincer's method

The transient moisture diffusion process in solid objects drying is similar to the heat transport process in these objects. The Fick diffusion law is exactly in the same form as the Fourier law of heat transfer, in which temperature and thermal diffusivity are replaced by moisture concentration and moisture diffusivity respectively. So, similar to the transient heat transfer analysis, we consider three situations for transient moisture diffusion. That is, the case the Biot number takes values as Bi≤0.1, 0.1<Bi<100, and Bi≥100 (Dincer 1998). For the case as Bi≤0.1, there is a negligible internal resistance to the moisture diffusivity within the lumber, which is almost impossible in wood drying process. So it will not be considered here. Cases, such as 0.1<Bi<100 and Bi≥100, representing infinite internal - surface resistances and negligible surface resistances respectively, are the two common situations in wood drying practice. We therefore mainly consider these two cases in present analysis.

The one-dimensional transient moisture diffusion equation in one-dimensional rectangular coordinate can be described by Fick's second law, which has the following compact form:

$$\frac{\partial M}{\partial t} = \frac{\partial}{\partial x}\left(D \cdot \frac{\partial M}{\partial x}\right) \tag{1}$$

where, M is the moisture content of wood (%), t the time (h), D the diffusion coefficient ($cm^2 \cdot h^{-1}$), and x is the space coordinate measured from the center of the board.

Equation (1) is subject to the following initial and boundary conditions:

$$\phi(x,0) = \phi_i = (M_i - M_e) \quad \text{for } 0.1<Bi<100, \text{ and } Bi \geq 100 \tag{2}$$

$$\left(\frac{\partial}{\partial t}\phi(0,t)\right) = 0 \quad \text{for } 0.1<Bi<100, \text{ and } Bi \geq 100 \tag{3}$$

$$-D\left(\frac{\partial}{\partial x}\phi(X,t)\right) = k \cdot \phi(X,t) \quad \text{for } 0.1<Bi<100, \quad \phi(X,t) = 0 \text{ for } Bi \geq 100 \tag{4}$$

where, X=L(thickness of specimens) , $\phi = (M - M_e)$, M_i is the initial moisture content, and M_e is the equilibrium moisture content.

The moisture content at a position of the solid wood is non-dimensionalized by the following equation:

$$E_d = \frac{(M - M_e)}{(M_i - M_e)} \tag{5}$$

The following dimensionless parameters are introduced for non-dimensional solution:

$$Bi = \frac{k \cdot X}{D} \tag{6}$$

$$Fo = \frac{D \cdot t}{X^2} \tag{7}$$

$$\Gamma = \frac{x}{X} \tag{8}$$

where, B_i is the Biot number for moisture loss, F_o the Fourier number for moisture loss, and Γ the dimensionless distance.

The simplified solution to the equation (1) is,

$$E_d = A \cdot B \tag{9}$$

where

$$A = \exp\left[\frac{(0.2533\ Bi)}{(1.3 + Bi)}\right] \tag{10}$$

$$B = \exp\left(-\mu^2 \cdot Fo\right) \tag{11}$$

The characteristic equation for root (μ) is,

$$\mu = \arctan(0.640443Bi + 0.380397) \quad \text{for } 0.1<Bi<100 \tag{12}$$

$$\mu = \left(\frac{\pi}{2}\right) \quad \text{for } Bi \geq 100 \tag{13}$$

where, μ is root of the characteristic equation.

From drying practice, we know that the dimensionless moisture content distribution can be regressed by an exponential function with the following form,

$$E_d = J \cdot \exp(-S \cdot t) \tag{14}$$

By taking A=J,

$$\left(\mu^2 \cdot Fo\right) = (S \cdot t) \tag{15}$$

where S is the drying coefficient (1/h), E_d the dimensionless moisture content for Dincer method and J is the lag factor.

From the definition of the Fourier number, the moisture diffusion coefficient can be obtained in the following form:

$$D = \left(\frac{S \cdot X^2}{\mu^2}\right) \tag{16}$$

Equation (16) can be used to determine the moisture diffusivity values for lumber subjected to drying, with parameters (Bi) and μ being extracted from Eqs. (10) and (12) respectively.

The equation determining the moisture transfer coefficients (k) can be written in the following forms:

$$k = \left(\frac{D}{X}\right) \cdot \left[\frac{(1 - 3.94813 \cdot \ln J)}{(5.13257 \cdot \ln J)}\right] \quad (17)$$

The solution procedures for wood moisture diffusivity and moisture transfer coefficient models are as follows.

1. The average moisture contents of lumber against time are determined experimentally and then non-dimensionalized using Eq. (5).
2. The dimensionless moisture content are regressed against time in the exponential form of Eq. (14), the lag factor (J) and drying coefficient (S) are determined accordingly.
3. The Biot number is determined from Eq. (10) by using the lag factor.
4. The root (μ) is calculated from Eq. (12).
5. The moisture diffusion coefficient is determined by using Eq. (16).
6. The moisture transfer coefficient is determined by using Eq. (17).

Crank's method

In this method, the governing model equation, initial and boundary conditions are similar to that of first method. By assuming the diffusion coefficient is independent of moisture content, the following equation for diffusion coefficients can be used (Siau 1984):

$$D = \frac{(E_c)^2 \cdot 4L_c^2}{5.1 \cdot t} \quad (18)$$

where, D is the diffusion coefficients, L_c is the half thickness of the board which is the diffusion direction, and t is the drying time. E_c is the fractional changes in average moisture content at time (t), can be determined by following equation:

$$E_c = \frac{(M - M_i)}{(M_e - M_i)} \quad (19)$$

According to Eq. (18), a plot of E_c^2 versus (t) are linear with a slope of 5.1 D/L_c^2, so D can be calculated from the slope of a linear regression equation of the experimental data, D can be calculated as follows:

$$D = \frac{4L_c^2}{5.1} \times Slope \quad (20)$$

where, $Slope = \frac{(E_C)^2}{t}$ and the moisture content (M) for the diffusion coefficient can be determined as $M = M_e + \frac{2}{3}(M_i - M_e)$.

Results and discussion

Diffusion coefficients determined by Dincer's method (Dincer 1995)

The dimensionless experimental moisture content distribution and the dimensionless moisture content variation from mathematical regression analysis are shown in Fig. 1. For experimental conditions of 50% RH at 50ºC, the lag factor and drying coefficient are 1.06 and 0.0234 1/h, respectively, while the determination coefficient (R^2) is 0.891. As J=1.06, this indicates that the Biot number is higher than 0.1, from Eq. (10) we know that Bi equals 0.4049. So for this experiment the valid criterion is $0.1<Bi<100$. Then the root value (μ) is deduced to be 0.569098. Having these data in hand, the moisture diffusion coefficient is found to be 0.072251 $cm^2 \cdot h^{-1}$ in Eq. (16) and the moisture transfer coefficient is determined to be 0.089219 $cm \cdot h^{-1}$ by using Eq. (17).

The experiment conditions' parameters and the other experimental values are shown in Table 1 and 2, respectively. In Table 2, the moisture diffusion coefficients have a increasing trend with condition temperature's increasing, and the relationships between D and Temperature are similar more or less to that with *Arrhenius* equation $D = 0.07 \cdot e^{\left(\frac{-(9,200 - 70M)}{R \cdot T}\right)}$ (cal/mole), where M is moisture content and R is air universal constant and T is temperature in Kelvin. However, the detailed description of the relationship between D and T for larch wood need to make in future, which should be the objective of further research.

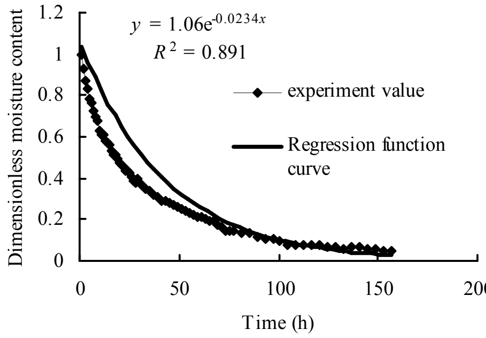

Fig. 1 Measured and regressed dimensionless moisture content for larch board dried at 50ºC and relative humidity of 50%.

Table 1. The condition parameters of experiments

Experiment no.	Temperature(℃)	Relative humidity (%)
1	50	50
2	60	50
3	70	50
4	85	50

Notes: *----Dried from wood FSP; air velocity was 0.5m/s approximately.

Table 2. Results of moisture diffusion coefficients and moisture transfer coefficients

Experiment no.	Moisture diffusion coefficient ($cm^2 \cdot h^{-1}$)	Moisture transfer coefficient ($cm \cdot h^{-1}$)
1	0.072251	0.089219
2	0.088445	0.09695
3	0.093091	0.052622
4	0.149985	0.193067

Diffusion coefficients determined by Crank's method

Moisture diffusion coefficients of different conditions are listed in Table 3. Because of the differences in calculation procedures,

the values of diffusion coefficients are all smaller than those of Dincer's method, but the relationships between D and T in Crank' method are similar to that of formers. With condition temperature's increasing, the diffusion coefficients increase, too.

Table 3. Average moisture diffusion coefficients calculated by Crank's method

Experiment no.	Moisture contents (%)	Diffusion coefficients ($cm^2 \cdot h^{-1}$)
1	18	0.0106
2	16	0.014
3	15	0.0216
4	12	0.026

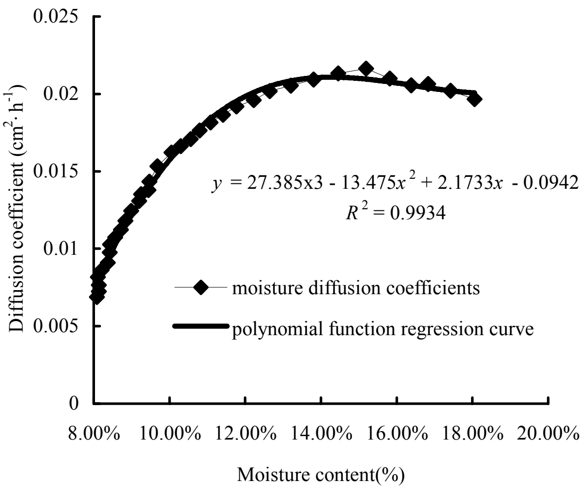

Fig. 2 Measured and regressed dynamic diffusion coefficients under condition with 70ºC, 50% of RH

The function relationship between diffusion coefficients (D) and moisture content (M) is represented in Fig. 2. Below wood FSP, the moisture diffusion coefficients first decrease somewhat slowly and then decrease acutely in great speeds. This changing trend can be regressed very well by using polynomial function; the determination coefficient (R^2) is 0.9937. The changing form under other experimental conditions is similar to the above one. The behavior of moisture diffusivity with changing moisture content can be explained by theory of the bound water transport in wood. As we know, the activation energy (E_b) of wood bound water increases with moisture content decreasing. The approximation expression of activation energy (E_b) is as follows.

$$E_b = 9{,}200 - 70M \ (cal/mol). \qquad (21)$$

where, M is the moisture content. As a result, with moisture content decreasing, more energy is needed to remove the bound water out of wood and difficulty for bound water movement increases accordingly. In macro level, the moisture diffusion coefficients decrease quickly with moisture content decreasing.

Conclusions

The moisture diffusion coefficients of larch wood lumber were calculated by using Dincer's method and Crank's method. Also the moisture transfer coefficients were determined by using Dincer's method. The results of diffusion coefficients from the first approach were higher than that of the second one generally. These results may be due to the differences between two analytical methods and also different characteristics between solid moisture diffusion process and heat transfer process. As a result, the solution procedures of relevant moisture diffusion differential equation need to be improved in the future.

All the experimental values indicate that moisture diffusivity of larch wood lumber increases with increasing of drying temperature, partly testifies the validity of *Arrhenius* equation, however the detailed study on relationship between D and Temperature as well as Relative Humidity is future need. In order to gain thorough analysis of both calculated moisture transfer coefficients and moisture diffusion coefficients which are determined by using the above two methods, dynamic moisture transfer and heat transfer tests with different specimen thickness are needed.

Below FSP, as moisture contents decrease, Larch wood diffusion coefficients decrease quickly due to the increase of bound water activation. The relationship between D and M can be regressed very well by using polynomial function.

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